UPR-738-T
HUB-EP-97/16
hep-th/9702205

BPS-Saturated Bound States of Tilted P-Branes in Type II String Theory

Klaus Behrndt[a] [1] **and Mirjam Cvetič**[b] [2]

[a] Humboldt-Universität, Institut für Physik
Invalidenstraße 110, 10115 Berlin, Germany

[b] Department of Physics and Astronomy
University of Pennsylvania, Philadelphia, PA 19104-6396

Abstract

We found BPS-saturated solutions of M-theory and Type II string theory which correspond to (non-marginally) bound states of p-branes intersecting at angles different from $\pi/2$. These solutions are obtained by starting with a BPS marginally bound (orthogonally) intersecting configurations of two p-branes (e.g, two four-branes of Type II string theory), performing a boost transformation at an angle with respect to the world-volume of the configuration, performing T-duality transformation along the boost-direction, S-duality transformation, and T- transformations along the direction perpendicular to the boost transformation. The resulting configuration is non-marginally bound BPS-saturated solution whose *static metric* possesses the off-diagonal term which *cannot* be removed by a coordinate transformation, and thus signifies an angle (different from $\pi/2$) between the resulting intersecting p-branes. Additional new p-branes are bound to this configuration, in order to ensure the stability of such a static, tilted configuration.

[1]e-mail: behrndt@qft2.physik.hu-berlin.de
[2]e-mail: cvetic@cvetic.hep.upenn.edu

1. Introduction and procedure

Recently a number of BPS-saturated configurations representing (non-marginally) bound states of various p-brane configurations in M- and Type II string theory were obtained.[3] In general such configurations are obtained by performing a subset of U-duality transformations on a marginally bound configuration, representing BPS states of (orthogonally intersecting) p-branes at a threshold, specified by independent harmonic functions for each p-brane [2]. Such solutions, obtained up to this point, are interpreted as bound state configurations of a set of p-branes whose pairs are either *parallel* to each-other or intersect *orthogonally*, and in some cases contain a wave along one of the p-brane world-volume direction. Thus, all the static solutions obtained in this class have *diagonal* internal metric.

The aim of this paper is to find static non-marginally bound BPS configurations with *non-diagonal* internal metric (which cannot be removed by a coordinate transformation). Such configurations therefore signify bound states of p-branes at *angles* different from zero and $\pi/2$. They are believed to be important when addressing higher dimensional embeddings of the generating solution for the four-dimensional BPS-saturated black holes (in toroidally compactified Type II string). This generating solution is parameterized by *five* charges [3, 4]; the four charges correspond to the marginally bound state of four orthogonally intersecting p-branes (e.g., $2 \perp 2 \perp 4 \perp 4$ in Type IIA string theory), while the fifth charge, which renders the solution non-marginally bound, is suspected to signify a tilting of the intersecting p-branes.[4]

The procedure we employ makes use of the following features of Type II string theory. All the p-brane solutions in ten dimensions are connected to each other by discrete (U) duality transformations. E.g., T-duality transformations transform all D-p-branes into one another and the S-duality converts D-branes into NS-NS sector p-branes (the fundamental string or 5-brane).

On the other hand, a p-brane can be rotated by making first a finite boost at an angle (to the world-volume of the p-brane), then setting the charge Q of the original p-brane to zero, and the boost parameter β to infinity, while keeping the product $Q \cdot \cosh^2 \beta$ finite. A resulting configuration is a fundamental string along the direction of the original boost. Again, by performing discrete U-duality transformations we can now create all other types of p-branes oriented along this direction.

In the case when the boost is taken finite such configurations have an interpretation as an interpolating, non-marginally bound state. Many of these (non-marginally) bound states, which are obtained by finite boosts, are discussed in the literature [6, 1, 7]. Note, however, that after the boost transformation such configurations become stationary ($G_{0m} \neq 0$). By performing T-duality transformations along the boost direction and a direction perpendicular to the boost, the resulting solution becomes *static*. So the boost parameter does not only create an additional brane, but it also rotates the whole configuration.

[3] For a recent extensive discussion of such configurations see Ref. [1] and references therein.
[4] From the D-brane world-volume perspective a related issue was addressed in Ref.[5].

As the last ingredient needed to obtain a tilted configuration is to perform a (continuous) S-duality ($SL(2,\mathbf{R})$ transformation between the two T-duality transformations. Such a transformation, while affecting the nature of the off-diagonal terms in the metric, it leaves the resulting configuration static, however, now with a non-diagonal internal metric.

We do the above sequence of transformations on a representative example of two (orthogonally) intersecting, 4-branes ($4 \perp 4$) with each of the 4-brane specified by a harmonic function.[5] This is a marginally bound BPS configuration which preserves 1/4 of supersymmetry.

On this initial configuration we then perform the following steps:

- Perform a boost β at an angle θ to the world-volume of the intersecting brane.
- Perform a sequence of T-S-T-duality transformations[6] , where the two discrete T-duality transformations are along the boost direction and a direction perpendicular to the boost. The (continuous) S-duality transformation is parameterized by an $SO(2)$ angle χ.
- As the last step we identify the constituent p-branes forming the bound state and the angles between them.

This procedure is described in Figure 1. Few additional comments are in order. After the first step, we have a stationary solution. Recall, that in order to obtain a static solution we have to perform T-duality twice along two orthogonal directions in the rotation plane. The resulting bound state consists not only of two 4-branes, but also contains additional p-branes. However, such a resulting configuration still contains only perpendicular and parallel p-branes. Again, in order to get a configuration where the branes are tilted with respect to each other we have to include a continuous S- transformation in between. Note also, that since we started with a Type IIA string solution, after the first T-duality we have a Type IIB string solution, and the S-duality mixes the fundamental string part with a D-string part.

2. Explicit transformations

The starting point is the (orthogonal) intersection of two 4-branes (intersecting at a 2-brane), with the following form of the ten-dimensional metric, the anti-symmetric (R-R)

[5]We choose that type of the starting configuration for the simplicity of obtaining the final structure of the metric. Note that the configuration $2 \perp 4$ is also of special interest because it is expected that the resulting tilted configuration of a 2-brane and 4-brane may naturally allow for an interpretation of a special case $2 \subset 4$[7] which preserves 1/2 of supersymmetry. Note also, that all such ten-dimensional configurations can be lifted to eleven-dimensions, thus obtaining analogous configurations of M-theory.

[6]This solution generating technique has been used in different context, e.g. in [8].

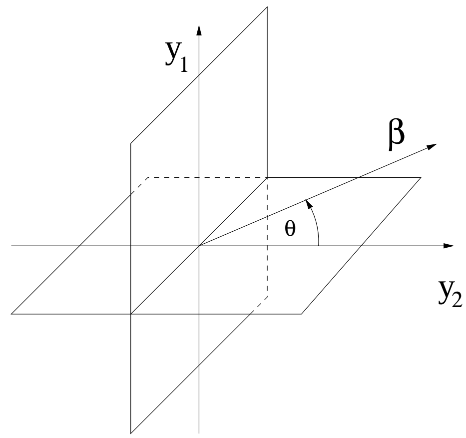

Figure 1: From the intersection of two 4-branes we pick out two world-volume coordinates (y_1 and y_2). The boost β is made under an angle θ. We T-dualize this configuration along both the boost direction and the direction orthogonal to the boost, while in between the two T-duality transformations we perform the S-duality transformations with an $SO(2)$ angle χ.

4-form field strength and the dilaton:

$$
\begin{aligned}
&ds_0^2 = \tfrac{1}{\sqrt{H_1H_2}}\,[(-dt^2 + H_1dy_1^2 + H_2dy_2^2) + dz_1^2 + dz_2^2 + H_1dy_3^2 + H_2dy_4^2 + H_1H_2d\vec{x}d\vec{x}] \\
&F^4 = (\ {}^*dH_1 \wedge dy_2 \wedge dy_4\ +\ {}^*dH_2 \wedge dy_1 \wedge dy_3) \wedge dt \wedge dz_1 \wedge dz_2 \\
&e^{-2\phi} = \sqrt{H_1H_2}\ .
\end{aligned}
\tag{1}
$$

and the harmonic functions $H_{1,2}$ are given by

$$
H_{1,2} = 1 + \frac{Q_{1,2}}{r}\ . \tag{2}
$$

with $r^2 = \vec{x}\vec{x}$ (with $\vec{x} = (x_1, x_2, x_3)$).

Coordinate transformations (boost at an angle)

We first transform the metric, and as the next step we shall discuss the R-R fields. Since we perform a boost β under an angle θ, we first rotate the (y_1, y_2) plane and make a boost along the new y_1 coordinate. If we denote the original metric by $G^{(0)}_{rs}$ $(r, s = t, y_1, y_2)$ the new metric G_{rs} is

$$
G = (\Omega_R\Omega_B)^T G^{(0)}(\Omega_R\Omega_B) = \frac{1}{\sqrt{H_1H_2}}\,(\Omega_R\Omega_B)^T \begin{pmatrix} -1 & & \\ & H_1 & \\ & & H_2 \end{pmatrix} (\Omega_R\Omega_B) \tag{3}
$$

with the rotation and boost transformation given by

$$\Omega_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \quad , \quad \Omega_B = \begin{pmatrix} \cosh\beta & -\sinh\beta & 0 \\ -\sinh\beta & \cosh\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{4}$$

The new metric can be written as

$$G_{rs} = \frac{1}{\sqrt{H_1 H_2}} \left(\eta_{rs} + \frac{Q_{rs}}{r} \right) \tag{5}$$

with

$$\begin{aligned} Q_{00} &= (Q_1 \cos^2\theta + Q_2 \sin^2\theta) \sinh^2\beta \ , & Q_{11} &= (Q_1 \cos^2\theta + Q_2 \sin^2\theta) \cosh^2\beta \ , \\ Q_{01} &= -(Q_1 \cos^2\theta + Q_2 \sin^2\theta) \sinh\beta \cosh\beta \ , & Q_{22} &= (Q_1 \sin^2\theta + Q_2 \cos^2\theta) \ , \\ Q_{02} &= -(Q_1 - Q_2) \sin\theta \cos\theta \sinh\beta \ , & Q_{12} &= (Q_1 - Q_2) \sin\theta \cos\theta \cosh\beta . \end{aligned} \tag{6}$$

The resulting metric is thus stationary with non-vanishing G_{01} and G_{02} components.

Duality transformations

By performing T-duality twice along the new y_1 and y_2 directions, we obtain a static metric, but with additional components of the anti-symmetric tensor.[7] Note that the starting configuration did not have an anti-symmetric tensor, i.e., $B_{rs} = B^{(0)}_{rs} = 0$. The new metric and the anti-symmetric tensor after the sequence of two T-duality operations is of the form:

$$\hat{G}_{rs} = \frac{1}{D_0} \begin{pmatrix} \det G & 0 & 0 \\ 0 & G_{22} & -G_{12} \\ 0 & -G_{12} & G_{11} \end{pmatrix} \quad , \quad \hat{B}_{rs} = \frac{1}{D_0} \begin{pmatrix} 0 & D_1 & -D_2 \\ -D_1 & 0 & 0 \\ D_2 & 0 & 0 \end{pmatrix} \tag{7}$$

with the sub-determinants of G_{rs} in (5): $D_0 = G_{11}G_{22} - G_{12}^2$, $D_1 = G_{10}G_{22} - G_{20}G_{12}$, $D_2 = G_{10}G_{21} - G_{11}G_{20}$. Using (3) we find: $\det G = \det G^{(0)} = -(\sqrt{H_1 H_2})^{-1}$. The non-vanishing time-like components of the anti-symmetric tensor indicate, that the new configuration contains a fundamental string.

Since we started with a Type IIA string solution, after the first T-duality we have a Type IIB solution. On this Type IIB solution we can perform a $SL(2, \mathbf{R})$ transformation and perform the second T-duality transformation afterwords. However, the $SL(2, \mathbf{R})$ transformation mixes the NS-NS fields with the R-R fields. Thus, before we display the final form of the metric and the anti-symmetric tensor, we first discuss the R-R gauge fields.

[7] An explanation of how T-duality along the boost leads to a fundamental string in the dual picture was given in [9].

By performing the rotated boost we obtain for R-R 4-form field strength in (1)

$$\begin{aligned} F^4 = &\ {}^*dH_1 \wedge dy_4 \wedge dz_1 \wedge dz_2 \wedge (-\sin\theta\, dt \wedge dy_1 + \cos\theta(\cosh\beta\, dt - \sinh\beta\, dy_1) \wedge dy_2) \\ &+ {}^*dH_2 \wedge dy_3 \wedge dz_1 \wedge dz_2 \wedge (\cos\theta\, dt \wedge dy_1 + \sin\theta(\cosh\beta\, dt - \sinh\beta\, dy_1) \wedge dy_2)\ . \end{aligned} \tag{8}$$

Next, after T-duality transformation along the y_1 direction we obtain a R-R-torsion and a 5-form field strength contribution. Using the transformations given in [10] we find

$$\begin{aligned} H_0^{R-R} &= \cosh\beta(\cos\theta\ {}^*dH_1 \wedge dy_4 + \sin\theta\ {}^*dH_2 \wedge dy_3) \wedge dt \wedge dy_1 \wedge dy_2 \wedge dz_1 \wedge dz_2 \\ F^5 &= dH_1 \wedge dz_1 \wedge dz_2 \wedge dy_4 \wedge (-\sin\theta dt + \cos\theta \sinh\beta dy_2) + \\ &\quad dH_2 \wedge dz_1 \wedge dz_2 \wedge dy_3 \wedge (\cos\theta dt + \sin\theta \sinh\beta dy_2) + (dual)\ . \end{aligned} \tag{9}$$

Since we started with a configuration of two 4-branes there is *no* R-R scalar for the Type IIB string solution. As the next step we perform the S-duality transformation, specified by an $SO(2)$ angle χ, which leaves F^5 invariant. The new NS-NS and R-R-torsions are given by

$$\begin{pmatrix} H^{NS-NS} \\ H^{R-R} \end{pmatrix} = \begin{pmatrix} \cos\chi\, H_0^{NS-NS} + \sin\chi\, H_0^{R-R} \\ -\sin\chi\, H_0^{NS-NS} + \cos\chi\, H_0^{R-R} \end{pmatrix}, \tag{10}$$

where H_0^{NS} is

$$H_0^{NS-NS} = d\frac{G_{12}}{G_{11}} \wedge dy_1 \wedge dy_2 + d\frac{G_{10}}{G_{11}} \wedge dy_1 \wedge dt \tag{11}$$

with G_{rs} defined in (5). Now, as the last step we perform the second T-duality along y_2 direction. Importantly, the R-R part in H^{NS-NS} *does not contribute to the new metric*, since it has no components in the y_2 direction.

Before further addressing the NS-NS fields we continue the discussion of the R-R fields. First, there is a non-trivial vector field part

$$F^2 = -\sin\chi\, d\frac{G_{12}}{G_{11}} \wedge dy_1\ . \tag{12}$$

However, there is no charge associated with this field strength, i.e. the corresponding integrals at spatial infinity vanish ($\int F = \int^* F = 0$). Therefore, there is *no* 0- or 6-brane contained in the resulting bound state configuration.

The 4-form gauge fields split into an electric and magnetic part. The electric part is given by

$$F^4 = -\sin\chi\, d\frac{D_1}{D_0} \wedge dy_1 \wedge dy_2 \wedge dt\ . \tag{13}$$

Hence, our configuration has to contain a two-brane, which couples to this field strength. The magnetic part can be written in terms of components as

$$\begin{aligned} F^4_{2\mu\nu\rho} &= \cos\chi H^{R-R}_{0\,\mu\nu\rho} \\ F^4_{\mu\nu\rho\lambda} &= F^5_{\mu\nu\rho\lambda 2} + \partial_{[\mu}\tfrac{G_{12}}{G_{11}} B^{R-R}_{0\,\rho\lambda]}\delta_{\nu 1}\ . \end{aligned} \tag{14}$$

Therefore, our final configuration is a bound state of two magnetic 4-branes, one electric two-brane (F^4_{0mnp}), a fundamental string (B^{NS-NS}_{0m}) and a NS-5-brane (coming from H_0^{R-R} in H^{NS-NS}, see (10)). Again there are no 0- or 6-branes.

Now we address the metric, the anti-symmetric tensor and the dilaton field. After the T-S-T transformations the first three components of the metric, the anti-symmetric tensor and the dilaton take the final form:

$$
\begin{array}{llll}
\hat{G}_{00} = \rho \frac{\det G}{D_0} \ , & \hat{G}_{11} = \frac{\cos^2\chi}{\rho}\frac{G_{22}}{D_0} + \frac{\sin^2\chi}{\rho} e^{-2\phi} \ , & \hat{G}_{22} = \frac{G_{11}}{\rho D_0} \ , & \hat{G}_{12} = -\frac{\cos\chi}{\rho}\frac{G_{12}}{D_0} \ , \\
\hat{B}_{10} = \cos\chi \frac{D_1}{D_0} \ , & \hat{B}_{20} = -\cos\chi \frac{D_2}{D_0} \ , & e^{-2\hat{\phi}} = e^{-2\phi}\frac{D_0}{\rho^3} &
\end{array}
\tag{15}
$$

with: $\rho^2 = \cos^2\chi + \sin^2\chi \, e^{-2\phi} G_{11}$ and χ is an angle of $SO(2) \subset SL(2, \mathbf{R})$. Inserting our metric and dilaton, ρ^2 defines a new harmonic function

$$
\hat{H} \equiv \rho^2 = 1 + \frac{Q_{11}\sin^2\chi}{r} \ . \tag{16}
$$

Also, using the metric (5) we can define two further harmonic functions $\tilde{H}_{1,2}$ by

$$
D_0 = \frac{\tilde{H}_1\tilde{H}_2}{H_1 H_2} \ . \tag{17}
$$

with the charges $\tilde{Q}_1$ and $\tilde{Q}_2$. Note that $\tilde{Q}_1\tilde{Q}_2 = Q_1 Q_2 \cosh^2\beta$, thus, $\tilde{H}_{1,2} = 1$ if $H_{1,2} = 1$.

Note, the metric (15) of the final configuration is *static* with the *off-diagonal* metric component which cannot be removed by a coordinate transformation, thus signifying the angle between constituent p-branes, different from $\pi/2$. Note also that non-zero components of the two-from field (15) signal the existence of the *fundamental* string.

Before discussing further the features of the resulting configuration we turn now to a discussion of a special cases.

3. A single four-brane bound state

We first consider the case of a single 4-brane. Thus we take the limit: $Q_2 = 0$, $Q_1 = Q$. As consequence $\tilde{H}_2 = 1$ and we obtain for the metric

$$
\begin{aligned}
d\hat{s}^2 &= \frac{\sqrt{H}}{\tilde{H}\sqrt{\hat{H}}} \Big[-\hat{H}dt^2 + ((1+\tfrac{Q_{22}}{r})\cos^2\chi + \tilde{H}\sin^2\chi)dy_1^2 + (1+\tfrac{Q_{11}}{r})dy_2^2 - \\
&\qquad -2\tfrac{Q_{12}}{r}\cos\chi \, dy_1 dy_2 \Big] + \sqrt{\hat{H}} d\bar{s}^2 \\
&= \frac{\sqrt{H}}{\tilde{H}\sqrt{\hat{H}}} \Big[-\hat{H}dt^2 + dy_1^2 + dy_2^2 + \tfrac{1}{r}\Big((Q_{22}\cos^2\chi + \tilde{Q}\sin^2\chi)dy_1^2 + Q_{11}dy_2^2 - \\
&\qquad -2Q_{12}\cos\chi \, dy_1 dy_2) \Big] + \sqrt{\hat{H}} d\bar{s}^2
\end{aligned}
\tag{18}
$$

where the $d\bar{s}^2$ denotes the unchanged part of the original metric (1). This metric can be diagonalized by a further rotation

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \to \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \ . \tag{19}$$

As result we get

$$d\hat{s}^2 = \sqrt{\frac{H}{\hat{H}}}\left[\frac{\hat{H}}{\tilde{H}}(-dt^2 + d\tilde{y}_2^2) + d\tilde{y}_1^2\right] + \sqrt{\hat{H}}d\bar{s}^2 \ , \tag{20}$$

and for the angle ψ we find

$$\tan\psi = (\,-\tfrac{Q_{22}}{Q_{12}\cos\chi}\,)_{Q_2=0} = -\frac{1}{\cot\theta\,\cosh\beta\,\cos\chi} \ . \tag{21}$$

There is a second solution given by the angle $\psi + \frac{\pi}{2}$. The harmonic functions are

$$H = 1 + \frac{Q}{r} \quad , \quad \tilde{H} = 1 + \frac{Q(\sinh^2\beta\ \cos^2\theta + 1)}{r} \quad , \quad \hat{H} = 1 + \frac{Q\sin^2\chi\ \cosh^2\beta\ \cos^2\theta}{r} \tag{22}$$

(recall: θ was the angle of the boost, β is the boost parameter and χ is the $SO(2) \subset SL(2,\mathbf{R})$ parameter). From the discussion of the gauge fields we know already, that this configuration contains the following branes

$$\begin{array}{lll} \text{4-brane :} & \hat{H} = 1\ ,\ \tilde{H} = H\ , & (\beta = \theta = \chi = 0) \\ \text{fundam. string :} & H = \hat{H} = 1\ , & (Q \to 0, \beta \to \infty;\ \sin\chi = 0) \\ \text{2-brane :} & H = 1\ ,\ \hat{H} = \tilde{H}\ , & (Q \to 0, \beta \to \infty;\ \cos\chi = 0) \\ \text{NS-5-brane :} & H = \hat{H} = \tilde{H}\ , & (\beta \to 0\ , \sin\chi\cos\theta = 1)\ . \end{array} \tag{23}$$

The solution (20) is therefore a bound state of these objects $(4 + 2 + 1_f + 5_{NS})$ and it breaks 1/2 of supersymmetry. For the special cases (23) the angle ψ is trivial; $\psi = 0, \frac{\pi}{2}$. But if all branes are turned on the whole configuration is rotated by the angle ψ with respect to the original location, (y_1, y_2) vs. $(\tilde{y}_1, \tilde{y}_2)$. However, the constituents of (20) are still perpendicular to each other.

4. The 4×4 bound state

Finally we return to the general solution obtained from considering *two* intersecting 4-branes (1), i.e. a configuration that breaks 1/4 of supersymmetry. In this case the T-S-T transformed metric is of the form (15) and can be cast in the form:

$$\begin{aligned} d\hat{s}^2 = \tfrac{\sqrt{H_1 H_2}}{\tilde{H}_1\tilde{H}_2\sqrt{\hat{H}}} \ & \Big[-\hat{H}dt^2 + \Big((1 + \tfrac{Q_{22}}{r})\cos^2\chi + \tilde{H}_1\tilde{H}_2\sin^2\chi\Big)\Big)dy_1^2 + (1 + \tfrac{Q_{11}}{r})dy_2^2 - \\ & -2\tfrac{Q_{12}}{r}\cos\chi\, dy_1 dy_2\Big] + \sqrt{\hat{H}}d\bar{s}^2 \ . \end{aligned} \tag{24}$$

In comparison to the former case (18) there is now a main difference. Before, we could factorize the dependence on the radius r (second eq. in (18)) so that by a rotation we could get rid of the off-diagonal part. However, now this is not possible, because of the $\tilde{H}_1\tilde{H}_2$ part in (24). Only for the special cases that one of the harmonic functions $\tilde{H}_{1,2}$ is trivial, i.e. $\tilde{Q}_1\tilde{Q}_2 = Q_1Q_2\cosh^2\beta = 0$, or if $\sin\chi\,\cos\chi = 0$ we still have a static configuration. E.g. for $\cos\chi = 0$ we get

$$d\hat{s}^2 = \sqrt{\frac{H_1H_2}{\hat{H}}}\left[\frac{\hat{H}}{\tilde{H}_1\tilde{H}_2}(-dt^2 + dy_2^2) + dy_1^2\right] + \sqrt{\hat{H}}d\bar{s}^2 \tag{25}$$

which is a bound state of $4\times 4 + 2 + 5_{NS}$. Note, that $\cos\chi = 0$ turns off the fundamental string, i.e. $B_{0r} = 0$ (see (15)). Again, in this case all objects are orthogonal. Since the first part is completely symmetric with respect to $Q_1 \leftrightarrow Q_2$, both 4-branes are now parallel in the (y_1, y_2) directions, but still orthogonal in the (y_3, y_4) directions, see (1). So we have rotated both 4-branes into each other, at least partly.[8]

In the case $Q_2 = 0$ we got the solution (20). On the other hand, if $Q_1 = 0$ we find the solution with the same structure

$$d\hat{s}^2 = \sqrt{\frac{H}{\hat{H}}}\left[\frac{\hat{H}}{\tilde{H}}(-dt^2 + d\tilde{\tilde{y}}_2^2) + d\tilde{\tilde{y}}_1^2\right] + \sqrt{\hat{H}}d\bar{s}^2 \tag{26}$$

but now with a new angle

$$\tan\psi' = \left(\tfrac{Q_{11}\cos\chi}{Q_{12}}\right)_{Q_1=0} = -\tan\theta\,\cosh\beta\,\cos\chi\ . \tag{27}$$

As before there is a second angle with an additional rotation of $\frac{\pi}{2}$.

How about the general case with charges Q_1 and Q_2 non-zero? The location of the two 4-branes can be determined by setting one of the charges to zero. We saw, that our procedure rotated both 4-branes, one by ψ and the other by ψ'. As a result, both 4-branes are now under an angle: $\frac{\pi}{2} - \psi + \psi'$ (see Figure 2). This tilted state is however only stable due to additional p-branes, in general a 2-brane, a fundamental string and an NS-5-brane. But by setting $\beta = 0$ ($D_1 = D_2 = 0$) we can turn off the 2-brane and the fundamental string and keep only the NS-5-brane. Note, the electric charges in our configuration are a consequence of the boost transformation. In special limits (typically if $\sin\chi\,\cos\chi = 0$ or $\beta \to \infty$) we recover an orthogonal configuration.

There is yet another way of interpreting our configuration. The y^1 and y^2 direction defines a deformed torus. Compactifying along these coordinates, yields a complex scalar field U which parameterizes the two-torus. The off-diagonal term of the metric has the consequence that U contains an imaginary-axion part. This axion part is also responsible that the two circles of the torus do not intersect orthogonally. So, the branes that are wrapped around these two circles intersect now at an angle different from $\pi/2$. By the T-S-T operation we had mapped a non-trivial S-modulus on the Type IIB side to an non-trivial U modulus (=tilting of the internal space) on the Type IIA side. Thus from the duality point of view the tilting of the branes was the expected result.

[8]Note however, that this configuration still preserves only 1/4 of supersymmetry.

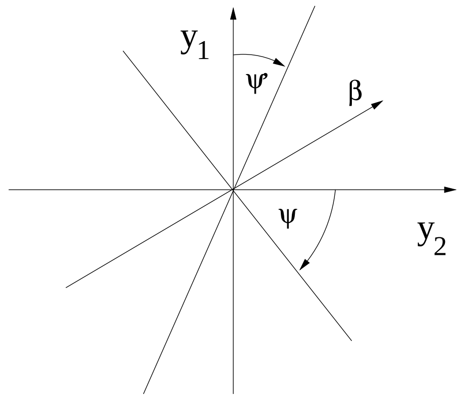

Figure 2: After making a finite boost between both four-branes and performing T-S-T-duality, the two 4-branes are tilted with respect to each other.

5. Conclusions

In this paper we considered an intersection of two four-branes and made a finite boost between the world-volumes of the branes (see Figure 1). The resulting configuration has a stationary metric ($G_{0m} \neq 0$). By T-dualizing these off-diagonal directions, the metric becomes static, but *not* diagonal. This new configuration is a (non-marginal) bound state of two four-branes, a two-brane, a fundamental string and an NS-5-brane ($4 \times 4 + 2 + 1_f + 5_{NS}$) , which has 1/4 of unbroken supersymmetry. In the single four-brane limit (see Section 3), it is possible to diagonalize the metric. In this case our procedure rotated the original four-brane and added a membrane, a fundamental string and an NS-5-brane. However, for the 4×4 case (see Section 4), this is not possible. By comparing the positions of the new four-branes, we find that generically they are not orthogonal to each other. Both branes have been rotated but with a different angle and therefore they intersect each other at an angle different from $\pi/2$. From the angles in (21) and (27) we see, that both have the same sign and thus both branes are rotated in the same direction. Note, also that for arriving at this conclusion it was essential that we made a S-duality between both T-duality transformations. Had the S-duality not been included, the resulting configuration would still have been orthogonal.

The case of the two instersecting four-branes is only an example and the procedure described in this paper is quite general. It can be straightforwardly adapted [11] to other intersecting brane configurations with the qualitatively same result, that the (static) branes are rotated by different angles.

Acknowledgements

We would like to thank A. A. Tseytlin for many helpful discussions and suggestions, and M. Costa for comments. The work is supported by U.S. DOE Grant Nos. DOE-EY-76-02-3071 (M.C.), the National Science Foundation Career Advancement Award No. PHY95-12732 (M.C.), the NATO collaborative research grant CGR No. 940870 (M.C.) and the

Deutsche Forschungsgemeinschaft (K.B.). K.B. thanks the University of Pennsylvania and M.C. the Humboldt University for the hospitality during visits where this work was initiated.

References

[1] A.A. Tseytlin, *Composite BPS configurations of p-branes in 10 dimensions and 11 dimensions*, `hep-th/9702163`.

[2] G. Papadopoulos and P.K. Townsend, *Intersecting M-branes*, `hep-th/9603087`; A.A. Tseytlin, *Harmonic superpositions of M-branes* `hep-th/9604035`; M. Cvetič and A. Sen, unpublished; K. Behrndt, E. Bergshoeff and B. Janssen, *Intersecting d-branes in ten-dimensions and six-dimensions*, `hep-th/9604168`; J.P. Gauntlett, D.A. Kastor and J. Traschen, *Overlapping branes in M-theorie*, `hep-th/9604179`; V. Balasubramanian and F. Larsen, *On D-branes and black holes in 4 dimensions* `hep-th/9604189`.

[3] M. Cvetič and A.A. Tseytlin, *Solitonic strings and BPS saturated dyonic black holes*, `hep-th/9512031`.

[4] M. Cvetič and C.M. Hull, *Black holes and U duality*, `hep-th/9606193`.

[5] M. Berkooz, M.R. Douglas and R.G. Leigh *Branes intersecting at angles*; `hep-th/9606139`, V. Balasubramanian and R. Leigh, *D-branes, moduli and supersymmetry*, `hep-th/9611165`.

[6] J.G. Russo and A.A. Tseytlin, *Waves, boosted branes and BPS states in M-theory*, `hep-th/9611047`; J.C. Breckenridge, G. Michaud and R.C. Myers, *More D-brane bound states*, `hep-th/9611174`; M.S. Costa, *Composite M-branes*, `hep-th/9609181`.

[7] J.M. Izquierdo, N.D. Lambert, G. Papadopoulos and P.K. Townsend, *Dyonic membranes*, `hep-th/9508177`.

[8] I. Bakas, *Space-time interpretation of S duality and supersymmetry violations of T duality*, `hep-th/9410104`.

[9] M.S. Costa and G. Papadopoulos, *Superstring dualities and p-brane bound states*, `hep-th/9612204`.

[10] E. Bergshoeff, C. Hull and T. Ortin, *Duality in type II superstring effective action*, `hep-th/9504081`; E. Bergshoeff and M. De Roo, *D-branes and T-duality*, `hep-th/9603123`.

[11] M. Cvetič and K. Behrndt, in preparation.